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A COMPARISON on DIFFERENT MATHEMATICAL MODELS to DESCRIBE the LACTATION CURVES in HOLSTEIN COWS

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Holştayn ineklerde laktasyon eğrisini tanımlamak için farklı matematiksel modellerin karşılaştırılması.

SUMMARY

Milk production records of 155 Holstein cows were analysed to find out the function providing the best fit for test day milk yields of first lactations. To accoplish this aim, five published equations to characterize individual lactation curves were compared. The equations were solved by applying multiple regression analysis. On the basis of R^2 values (coefficient of multiple determination), inverse polynomial ($t/y_t=b_0+b_1t+b_2t^2$, 96.2%) was the best choice. This function was followed by gamma-type ($\ln y_t=\ln a+b\ln t-ct$, 65.2%), linear hyperbolic ($y_t=\beta_0+\beta_1 t+\beta_2 t^2$,64.5%), parabolic exponential ($\ln y_t=\ln a+bt+ct^2$,63.1%), and quadratic ($y_t=a+bt+ct^2$,61.7%) functions.

KEY WORDS: Dairy cow, Holstein, Lactation curve, mathematical models.

ÖZET

Bu çalışma tanınmış beş matematiksel modeli, Batı Anadolu'da yetiştirilen Holştaynların periyodik süt verim kayıtlarına uygunluk yönünden karşılaştırmak ve inekleri laktasyon eğrisi özellikleri bakımından değerlendirirken gerçeğe en yakın sonuçları verecek fonksiyonları saptamak için yapılmıştır. Bu amaçla bir Türk-İtalyan işbirliği projesi olan ANAFI kapsamında 1990-1994 yılları arasında Türkiye'ye getirilmiş 155 ineğin birinci laktasyon kayıtları kullanılmıştır. Aylık süt kontrol günü verimlerine uygulanan fonksiyonların katsayıları çoklu regresyon analizi ile bulunmuştur. Çoklu belirleme katsayısı (R^2) değerlerine göre, ters polinomiyal fonksiyon($t/y_t=b_0+b_1t+b_2t^2$) en iyi uyumu (%96.2) sağlamış olup, bunu gama ($\ln y_t=\ln a+b\ln t-ct$,%65.2), doğrusal hiperbolik ($y_t=\beta_0+\beta_1 t+\beta_2 t^2$,%64.2), parabolik üstel ($\ln y_t=\ln a+bt+ct^2$,%63.1) ve kuadratik ($y_t=a+bt+ct^2$,%61.7) fonksiyonlar izlemiştir.

ANAHTAR KELİMELER: Süt ineği, Holştayn, Laktasyon eğrisi, matematiksel modeller.

INTRODUCTION

Milk yield is the most significant economic trait in dairy cattle. Milk production records were generally used to investigate the lactation traits of dairy cows. It is important that these data to give accurate information and to reflect differences among individual cows and treatment groups. To realize this, mathematical models can be applied to data from individual cows. The resultant lactation curves can then be used to estimate the lactation traits such as initial and peak yields, the ability of a cow to maintain her maximum production after attaining the peak yield (persistency), and daily or total milk yields (Batra et al. 1987, Grossman et al. 1986, Rao and Sundaresan 1982, Schneeberger 1981, Warona et al. 1998, Wood 1967, Wood 1969). But the multiple coefficient of determination (R^2) measuring how well the model fits the data differs from model to model (Hohenboken et al. 1992, Kolte et al. 1986, Malhotra et al. 1980, Pundir and Kaushik 1993, Sobrinho et al. 1986, Yadav et al. 1977).

In the present study five published lactation curve equations were compared to find out the best fitted function to explain the lacatation curves in Holstein Heifers.

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MATERIALS and METHODS

This study was carried out on the first lactation records of 155 Holstein cows. These cows were brought to Turkey from Italy in the scope of a Turkish – Italian collaboration project (ANAFI). They were delivered to Ankara, Aydin, Balikesir, Burdur, Denizli, Isparta, Izmir, Manisa, Mugla and Usak provinces. The following restrictions were made for cows to be included: first calving between 1990 and 1994, first test day prior to 15 days into lactation, first lactation finished and days of lactation more than 305 days, and pregnant with her second calf.

Five mathematical models, viz. parabolic exponential, inverse polynomial, gamma-type, quadratic, and linear hyperbolic functions were studied.

A brief descriptions of five functions used are as below:

1- Parabolic exponential function: This function was suggested by Robb (1940) and represented as follows:

$$y_t = ae^{(bt+ct^2)}$$

In its logarithm form, this is reduced to

$$\ln y_t = \ln a + bt + ct^2$$

2- Inverse polynomial function: Nelder (1966) described a family of inverse polynomial curves of which one can be used in dairy studies as a satisfactory model. This is

$$y_t = t(b_0 + b_1 t + b_2 t^2)^{-1}$$

Where y_t is the yield in time t and b_0, b_1, and b_2 are constant to be estimated.

The above equation after simplification becomes:

$$\frac{t}{y_t} = b_0 + b_1 t + b_2 t^2$$

3-Gamma-type function: Wood (1967) described the following equation for the lactation curve:

$$y_i = a t_i^b e^{-ct}$$

Where y_i is the milk yield, ti the day in lactation at ith recording, and a,b,c are constants.

After logarithmic transformation, the model can be expressed in the following form:

$$\ln y_i = \ln a + b \ln t - ct$$

4-Quadratic function: Malhotra et al. (1980) used a number of functions to explain the lactation curve and one of them was the following equation:

$$y_t = a + bt + ct^2$$

5- Linear hyperbolic function: The following function was proposed by Bianchini (1984) to describe a single lactation and an estimate of total production:

$$y_t = \beta_0 + \beta_1 t + \beta_2 \frac{1}{t}$$

Where y_t is milk production at time t, β_0, β_1, β_2 are parameters to be estimated. The constants of inverse polynomial and linear hyperbolic functions have been shown as a, b,and c constants in Table 1.

Monthly recorded test day milk yields were regressed on time (day) by linear multiple regression and the constants were determined. The coefficients of multiple determination were estimated for each function along with $S_{y.x}$ (Standard error of the estimate) by using conventional statistical methods (Oltman and Lackritz 1991, Steel and Torrie 1980).

RESULTS

Estimates of constants a, b, c of five fitted functions along with their respective R^2 and $S_{y.x}$ values were calculated and presented in Table 1.

Table 1: Mean Estimates of Constants of Five Fitted Functions, R^2 and $S_{y.x}$ Values Based on Monthly-recorded Test Day Milk Yields.

Type of function	Estimates				
	a	b	c	R^2	$S_{y.x}$
Parabolic exponential	20.57777±0.457534	0.00083±0.000312	-0.00001±0.000001	0.631	0.135
Inverse polynomial	0.65916±0.126679	0.02310±0.004081	0.00019±0.0000220	0.962	1.646
Gamma type	16.53513±0.740688	0.14722±0.015749	0.00307±0.000194	0.652	0.132
Quadratic	20.80657±0.441091	0.00602±0.005174	-0.00011±0.000017	0.617	2.399
Linear Hyperbolic	24.69973±0.507506	-0.03701±0.001955	-44.66557±5.390831	0.645	2.302

DISCUSSION

As regards R^2 values, inverse polynomial function had the highest value. So among all the five functions used this function describe the variability to the extend of 96.2%. Inverse polynomial function were followed by gamma-type (65.2%), linear hyperbolic (64.5%), parabolic exponential (63.1%), and quadratic (61.7%) functions. Kolte et al. (1986) found lower R^2 values for the above-mentioned models in Sahiwal cows, but similar results for inverse polynomial (90.8%) and gamma (68.5%) functions were obtained by Batra et al. (1987) in Canadian Holsteins. Yadav et al. (1977) reported that the inverse polynomial function explained 99% of the variability and gamma function accounted for 95%-98% of the variability in Hariana and Hariana X Friesian crosses. Kolte et al. (1986) and Batra et al. (1987) used weekly and average daily milk yields in a week respectively. But Yadav et al. (1977) fitted the models on the average weekly milk yields pooled over lactations and farms rather than monthly test day observations that were used in the present study. The higher or lower R^2 values of same function may be due to fitting of models on different kind of data and breeds in different studies.

CONCLUSIONS

The results of this study indicate that the inverse polynomial function provided a better fit to the lactation curves of Holstein heifers than the other five functions based on comparison of R^2 values. Gamma-type and Linear Hyperbolic functions can be considered as the second and third best fitted ones. These functions may be preferred in cow evaluations for different lactation traits and each of them subjected to more detailed studies by using different regression methods and cattle breeds.

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